Nonlinear electrostatic waves in a magnetized plasma

Chen Yinhua and Lu Wei
Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230027, People's Republic of China

M. Y. Yu
Institut für Theoretische Physik, Ruhr-Universität Bochum, D-44780 Bochum, Germany

(Received 8 January 1999)

The modulational interaction of finite-amplitude high-frequency electrostatic waves propagating at an arbitrary angle to an external magnetic field with slow plasma motion is considered. A set of nonlinear evolution equations describing the interaction is obtained. New types of solitary waves propagating at nearsonic speeds are found. [S1063-651X(99)10408-2]

PACS number(s): 52.35.Mw, 52.35.Fp, 52.35.Sb, 52.35.Hr

I. INTRODUCTION

Wave modulation and occurrence of localized electric fields and density depletions or enhancements in plasmas have been extensively investigated over a period of more than two decades [1–7]. Because of the many observations of wave modulation in applicational [8] and space [9–12] plasmas, recently there has been renewed interest in the nonlinear behavior of electrostatic waves in magnetized plasmas. Several authors have considered the interaction of high-frequency (hf) electrostatic waves with low-frequency (lf) oscillations in a magnetized plasma [13–17]. In particular, it has been found that the modulation of perpendicularly (to the ambient magnetic field) propagating upper-hybrid waves by acoustic type lf waves can lead to the appearance of smooth as well as cusped wave packets. It was also found [17] that the modulation of lower-hybridlike waves by Alfvén-like waves can also lead to similar results. However, in all such studies only exactly or nearly parallel or perpendicular propagation of the hf waves was considered. It is thus of interest to study the properties of nonlinear waves propagating in arbitrary directions.

In this paper, we consider the slow modulation of hf electrostatic electron waves propagating at an arbitrary angle to the external magnetic field. The evolution equations governing the nonlinear coupling of the high- and low-frequency waves are obtained. It is found that quasistationary sub and nearsonic localized wave envelopes can propagate in all directions, but their profiles depend on the angle of propagation as well as the other plasma parameters. New types of nearsonic solitary wave solutions are found.

II. BASIC EQUATIONS

We consider hf electrostatic waves in a magnetized plasma. The ambient magnetic field $\mathbf{B}_0$ is along the z axis. The fluid equations governing the motion of the electrons and ions are

$$\partial_t n_\alpha + \nabla\cdot(n_\alpha \mathbf{u}_\alpha)=0, \quad (1)$$

$$\partial_t \mathbf{u}_\alpha + \mathbf{u}_\alpha\cdot\nabla \mathbf{u}_\alpha = \frac{q_\alpha}{m_\alpha}\left(\mathbf{E}+\frac{1}{c}\mathbf{u}_\alpha\times\mathbf{B}_0\right)-\frac{\gamma_\alpha T_\alpha}{n_\alpha}\nabla n_\alpha, \quad (2)$$

$$\nabla\cdot\mathbf{E}=4\pi q_\alpha(n_i-n_e), \quad (3)$$

where the subscripts $\alpha=e,i$ stand for the electrons and ions, respectively.

We assume that the electric field $\mathbf{E}^h$ for the hf wave lies in the x,z plane, and that the angle between the electric field $\mathbf{E}^h$ and the constant external magnetic field $\mathbf{B}_0$ is $\theta(0\leqslant\theta\leqslant\pi/2)$. The hf field may be expressed as

$$\mathbf{E}^h=-\nabla\Phi^h=\tfrac{1}{2}\mathbf{E}(x,z,t)\exp(-i\omega_0 t)+\text{c.c.}, \quad (4)$$

where ω_0 is the frequency and $\mathbf{E}(x,z,t)$ is the amplitude of the hf (or pump wave) electric field. Because of nonlinear interactions the wave amplitude varies slowly in time and space. We can thus write $n_e=n_0+n_e^l+n_e^h$, $n_i=n_0+n_i^l$, $\mathbf{u}_e=\mathbf{u}_e^l+\mathbf{u}_e^h$, and $\mathbf{E}=\mathbf{E}^l+\mathbf{E}^h$, where n_0 denotes the number density of the plasma in equilibrium. The subscripts l and h stand for the high- and low-frequency perturbations, respectively. For modulational interactions, the hf and lf regimes are widely separated, so that the lf response appears as a modulation of the equilibrium or background quantities [1].

Substituting Eq. (4) into Eqs. (1)–(3), we obtain the equations describing the hf motion

$$\partial_t n_e^h + \partial_x(n u_{ex}^h)+\partial_z(n u_{ez}^h)=0, \quad (5)$$

$$\partial_t \mathbf{u}_e^h = -\frac{e}{m_e}\left(\mathbf{E}^h+\frac{1}{c}\mathbf{u}_e^h\times\mathbf{B}_0\right)-\frac{u_{te}^2}{n}\nabla n_e^h, \quad (6)$$

$$\nabla\cdot\mathbf{E}^h=-4\pi e n_e^h, \quad (7)$$

where $u_{te}=(\gamma_e T_e/m_e)^{1/2}$ is the thermal velocity of electrons and $n=n_0+n_e^l$ is the slowly modulated background electron density.

In general, the direction of the lf modulation can be different from that of wave propagation. For optimum coupling, it is expected that the modulation should be along the propagation direction, say, at an angle θ to $\mathbf{B}_0$. Thus, without loss of generality we can assume that the waves propagate in the x,z plane. Substituting Eq. (4) into Eqs. (5)–(7), we obtain for the modulated hf waves,

$$(\partial_t^4+\omega_{ce}^2\partial_t^2)\nabla^2\Phi^h+\omega_{pe}^2\mathcal{L}_1\Phi^h=\omega_{pe}^2\mathcal{L}_2\Phi^h, \quad (8)$$

where $\nabla^2=\partial_x^2+\partial_z^2$, $\omega_{pe}=(4\pi n_0 e^2/m_e)^{1/2}$, and $\omega_{ce}=eB_0/m_e c$ are the electron plasma and gyro frequencies, respectively. We have also defined

$$\mathcal{L}_1=(\nabla^2\partial_t^2+\omega_{ce}^2\partial_z^2)(1-\gamma_e\lambda_{De}^2\nabla^2)$$

and

$$\mathcal{L}_2=\{\partial_t^2[\partial_x(N\partial_x)+\partial_z(N\partial_z)]+\omega_{ce}^2\partial_z(N\partial_z)\}(\gamma_e\lambda_{De}^2\nabla^2-1),$$

where $\lambda_{De}=(T_e/4\pi n_0 e^2)^{1/2}$ is the Debye length and $N=n_e^l/n_0$ is the normalized electron perturbation density. When the nonlinear coupling term on the right-hand side of Eq. (8) is neglected, the latter one describes linear hf ($\partial_t>\omega_{pe},\omega_{ce}$) electrostatic waves propagating obliquely to the external magnetic field.

The lf motion consists of obliquely propagating generally mixed ion-acoustic and ion-cyclotron waves driven by the ponderomotive force of the hf waves. The governing equations are then

$$\partial_t n_\alpha+\partial_x(n_\alpha u_{\alpha x})+\partial_z(n_\alpha u_{\alpha z})=0, \tag{9}$$

$$m_\alpha\partial_t\mathbf{u}_\alpha=q_\alpha\left(-\nabla\varphi+\frac{1}{c}\mathbf{u}_\alpha\times\mathbf{B}_0\right)-\frac{T_\alpha}{n_\alpha}\nabla n_\alpha+\mathbf{F}_{p\alpha}, \tag{10}$$

where $\alpha=e,i$ and φ is the lf perturbation potential. For simplicity, we have dropped the superscript l for the lf quantities. We shall also use the quasineutrality condition $n_i=n_e$ valid for long wavelength lf perturbations. Furthermore, we have defined [14,15]

$$\mathbf{F}_{p\alpha}=-m_\alpha\langle\mathbf{u}_\alpha\cdot\nabla\mathbf{u}_\alpha\rangle, \tag{11}$$

which is the ponderomotive force acting on the charged particles by the nonlinear hf field with a slowly varying amplitude. They are obtained by averaging (over the fast time) of the nonlinear terms in the equations of motion [1–3]. Here the angular brackets denote averaging over the fast motion.

It is convenient to write $F_{\alpha x}=-\partial_x Q_{\alpha x}$ and $F_{\alpha z}=-\partial_z Q_{\alpha z}$, where

$$Q_{\alpha x}=\frac{q_\alpha^2}{4m_\alpha}\left[\frac{\omega_0^2|E_x^h|^2}{(\omega_0^2-\omega_{c\alpha}^2)^2}+\frac{|E_z^h|^2}{\omega_0^2-\omega_{c\alpha}^2}\right], \tag{12}$$

$$Q_{\alpha z}=\frac{q_\alpha^2}{4m_\alpha}\left[\frac{|E_x^h|^2}{\omega_0^2-\omega_{c\alpha}^2}+\frac{|E_z^h|^2}{\omega_0^2}\right], \tag{13}$$

where we have made use of the linear solutions of the hf electron equations (5) and (6).

Since the ponderomotive force is inversely proportional to the mass of the particles, we can ignore Q_{ix} and Q_{iz}. From Eqs. (9) and (10), assuming cold ions, we obtain the equations for the lf plasma response

$$(\partial_t^2+\omega_{ci}^2)\partial_t^2 N-c_s^2[\nabla^2\partial_t^2+\omega_{ci}^2\partial_z^2]\varphi=0, \tag{14}$$

$$\varphi=N+Q_{ez}, \tag{15}$$

where electron inertia in the lf ion-dominated motion has been neglected, the lf electrostatic potential φ has been normalized by T_e/e, and Q_{ex} and Q_{ez} have been normalized by T_e. Note that because the modulation is in the direction of the hf wave propagation, Q_{ex} does not contribute to the nonlinear coupling. Furthermore, although the lf waves are nonlinearly driven their amplitude remains small, so that self-nonlinearities are neglected.

From Eqs. (14) and (15) we easily obtain

$$[(\partial_t^2+\omega_{ci}^2-c_s^2\nabla^2)\partial_t^2-c_s^2\omega_{ci}^2\partial_z^2]N =c_s^2(\nabla^2\partial_t^2+\omega_{ci}^2\partial_z^2)Q_{ez}, \tag{16}$$

which describes electrostatic ion waves in the presence of the ponderomotive force. Note that for arbitrary angle of propagation the ion-acoustic and ion-cyclotron waves are linearly coupled.

Equations (8) and (16) describe the nonlinear modulation of hf electrostatic waves by lf motion in a magnetized plasma. As is the case for most such modulations, for the hf waves the dominant nonlinearity comes from their coupling to the lf density perturbations, and for the lf waves it is from the ponderomotive force of the hf waves acting on the electrons.

III. THE EVOLUTION EQUATIONS

In the following, we consider the modulated amplitude of the hf waves. It is convenient to define the dimensionless coordinate ξ such that $\rho_s\partial_x=\sin\theta\partial_\xi$, where $\rho_s=c_s/\omega_{ci}$ is the effective gyroradius. Separating the fast and slow motion [1–3] by letting $\partial_t\approx i\omega_0+\partial_\tau$, where $\partial_\tau\ll\omega_0$, we obtain from Eq. (8),

$$-2i\epsilon\partial_\tau\mathcal{E}+\mathcal{P}\mathcal{E}+\mathcal{Q}\partial_\xi^2\mathcal{E}=\kappa N\mathcal{E}, \tag{17}$$

where we have defined $\epsilon=\omega_0\omega_{ci}/\omega_{pe}^2=\mathcal{O}(m_e/m_i)\ll 1$, $\tau=\omega_{ci}t$, $\mathcal{E}=E/\sqrt{8\pi n_0 T_e}$,

$$\mathcal{P}=-\frac{\omega_0^4-\omega_0^2\omega_{uh}^2+\omega_{pe}^2\omega_{ce}^2\cos^2\theta}{\omega_{pe}^2(2\omega_0^2-\omega_{uh}^2)},$$

$\omega_{uh}=(\omega_{pe}^2+\omega_{ce}^2)^{1/2}$ is the upper-hybrid frequency, $\mathcal{Q}=\kappa v_A^2/c^2$, $v_A=\sqrt{B_0^2/4\pi m_i n_0}$ is the Alfvén speed, and $\kappa=(\omega_{ce}^2\cos^2\theta-\omega_0^2)/(2\omega_0^2-\omega_{uh}^2)<0$. We note that for weakly nonlinear modulation $\mathcal{P}\approx 0$, since the hf waves should satisfy the dispersion relation

$$\omega_0^4-\omega_0^2\omega_{uh}^2+\omega_{pe}^2\omega_{ce}^2\cos^2\theta=0 \tag{18}$$

in the linear limit. Here we keep it finite in order to allow for a small frequency shift, which may arise because of the modulation. For $\theta\approx 0$, one recovers the electron plasma waves, and for $\theta\approx\pi/2$ one recovers the upper-hybrid waves (here we do not consider resonances and cutoffs). In the latter case Eq. (17) reduces to that for upper-hybrid wave modulation [13–15].

For the lf density modulation, Eq. (16) yields

$$[(\partial_\tau^2-\partial_\xi^2+1)\partial_\tau^2-\cos^2\theta\,\partial_\xi^2]N=A(\partial_\tau^2+\cos^2\theta)\partial_\xi^2|\mathcal{E}|^2, \tag{19}$$

where $A\equiv\omega_{pe}^2(\omega_0^2-\omega_{ce}^2\cos^2\theta)/2\omega_0^2(\omega_0^2-\omega_{ce}^2)>0$. Equations (17) and (19) describe the evolution of the modulated hf electrostatic waves. The former is a nonlinear Schrödinger equation with the nonlinear term arising from the ponderomotive-force-driven density response, given by the latter equation. The equations are similar in structure to those describing upper-hybrid wave modulation [13–15]. From Eqs. (17) and (19) one can investigate the nonlinear evolution of modulational instabilities in the many possible parameter regimes. To our knowledge, there does not exist any general method of solution for these equations. For hf waves propagating perpendicularly ($\theta=\pi/2$) to the external magnetic field, Eq. (19) becomes

$$(\partial_\tau^2-\partial_\xi^2+1)N=\frac{1}{2}\frac{\omega_{pe}^2}{\omega_0^2-\omega_{ce}^2}\partial_\xi^2|\mathcal{E}|^2, \tag{20}$$

which is similar to the corresponding equation for upper-hybrid waves [15].

IV. MODULATION OF OBLIQUELY PROPAGATING WAVES

In general, Eqs. (17) and (19) are difficult to solve. Many authors have considered the limits of Langmuir ($\theta\approx 0$) [1–5] and upper-hybrid ($\theta\approx\pi/2$) [14–16] wave modulation by various lf perturbations. They derived the nonlinear evolution equations governing the slowly varying amplitude and found that under appropriate conditions steadily propagating smooth as well as cusped envelope solutions can exit.

We study the modulation of oblique propagation of hf electrostatic waves and look for quasistationary localized solutions. The latter are often considered to be the saturated states of the corresponding modulational instabilities. Accordingly, we let $N(\xi)=N(\eta)$ and $\mathcal{E}(\xi,\tau)=\mathcal{E}(\eta)\exp[i(\Theta\tau+\Gamma\xi)]$, where $\eta=\xi-M\tau$ and M is the speed (normalized by the ion-acoustic speed) of the localized wave packet. Equation (17) yields $\Gamma=-\epsilon M/\mathcal{Q}$ for the phase factor, and the equation for the amplitude $\mathcal{E}(\eta)$ of the wave envelope

$$\mathcal{Q}\partial_\eta^2\mathcal{E}(\eta)+\Delta\mathcal{E}(\eta)-\kappa N(\eta)\mathcal{E}(\eta)=0, \tag{21}$$

where $\Delta=\mathcal{P}+2\epsilon\Theta-\epsilon^2M^2/\mathcal{Q}$ is the total nonlinear frequency shift. The latter allows for departures from the linear hf wave frequency arising from the nonlinear coupling. Note that since $\mathcal{P}\approx 0$ and $\epsilon^2M^2/\mathcal{Q}\ll 1$, the main term in Δ is $2\epsilon\Theta$, which is an arbitrary constant introduced in the reduction of Eq. (17) to the ordinary differential equation (21), and will be determined by amplitude of the modulated waves.

From Eq. (19), we obtain

$$\begin{aligned}\{M^2[(M^2-1)\partial_\eta^2+1]-\cos^2\theta\}N(\eta)\\ =A(M^2\partial_\eta^2+\cos^2\theta)\mathcal{E}^2(\eta)\end{aligned} \tag{22}$$

for the density modulation. Note that $\Delta>0$, $\kappa<0$, $\mathcal{Q}<0$, and $A>0$.

Equations (21) and (22) are the general coupled nonlinear ordinary differential equations describing the amplitude of the quasistationary propagating hf electrostatic waves in a magnetized plasma. They can easily be integrated numerically for any of the many possible parameter regimes. Since the profile of the solution depends strongly on the relation between N and $\mathcal{E}$, and the parameters κ and A also contain θ; it is clear from Eq. (22) that the angle of wave propagation with respect to the external magnetic field is an important parameter in determining the profile of the solution. In the limit $\theta=0$, we recover the relation $N=A\mathcal{E}^2/(M^2-1)$ with $A=\omega_{pe}^2/2\omega_0^2$ for Langmuir wave modulation [1]. For $\theta=\pi/2$, we have the relation $[(M^2-1)\partial_\eta^2+1]N=A\partial_\eta^2\mathcal{E}^2$ with $A=\omega_{pe}^2/2(\omega_0^2-\omega_{ce}^2)$ for upper-hybrid wave modulation [15]. It is also of interest to note that for static response ($M=0$), one has $N=-A\mathcal{E}^2$ for any θ. So that static modulation of waves propagating at any angle to the external magnetic field physically behaves like that for the Langmuir waves. In general, the modulation leads to propagating enveloped wave. Localized solutions similar to the Langmuir and upper-hybrid solitary waves can exist for near-parallel or near-perpendicular wave propagation [13–16]. In the following we shall consider regimes, which have not yet been addressed in the literature.

For $\partial_\eta^2\ll 1$ and $M^2\partial_\eta^2\ll\cos^2\theta$, one obtains from Eq. (22),

$$N=\frac{A\cos^2\theta}{M^2-\cos^2\theta}\mathcal{E}^2(\eta), \tag{23}$$

which is of similar form to that of the density modulation by Langmuir waves. However, here the angle θ can strongly affect the magnitude of the density response. Substituting Eq. (23) into Eq. (21), one obtains

$$\mathcal{Q}\partial_\eta^2\mathcal{E}(\eta)+\Delta\mathcal{E}(\eta)-\frac{\kappa A\cos^2\theta}{M^2-\cos^2\theta}\mathcal{E}^3(\eta)=0, \tag{24}$$

describing the evolution of $E(\eta)$. Equation (24) has the localized solution

$$\mathcal{E}(\eta)=\mathcal{E}_0\operatorname{sech}(\eta/a), \tag{25}$$

where $\mathcal{E}_0=[\Delta(M^2-\cos^2\theta)/\kappa A\cos^2\theta]^{1/2}$ is the amplitude of the wave envelope, and $a=(-\mathcal{Q}/\Delta)^{1/2}$ is its width. Clearly, one must have $\Delta>0$, and the normalized speed of this solitary wave must satisfy $M^2<\cos^2\theta$.

We now present two solitary wave solutions, which seem not to have been discussed before. In the the nearsonic regime $M^2-1=\mathcal{O}(\epsilon)$, one gets from Eq. (22),

$$N(\eta)\approx\frac{AM^2}{M^2-\cos^2\theta}\partial_\eta^2\mathcal{E}^2+\frac{A\cos^2\theta}{M^2-\cos^2\theta}\mathcal{E}^2 \tag{26}$$

for the density modulation. Substituting Eq. (26) into Eq. (21) and integrating once, one obtains for the field amplitude $\mathcal{E}(\eta)$,

$$(d_{\tilde\eta}\mathcal{E})^2-\frac{\mathcal{E}^2-\mathcal{E}^4/\mathcal{E}_0^2}{1-\mathcal{E}^2/\mathcal{E}_\star^2}=0, \tag{27}$$

where $\tilde\eta=\eta/a$, $\mathcal{E}_\star^2=v_A^2(M^2-\cos^2\theta)/2AM^2c^2$, and for localized solutions, the integration constant has been set to zero. Note that $\mathcal{E}_\star^2$ and $\mathcal{E}_0^2$ can be negative and are of opposite signs.

The behavior of the solutions of Eq. (27) can easily be visualized by treating the latter as the energy integral of a classical particle in a potential well (the Sagdeev potential

[18]). Recalling $A>0$, $\kappa<0$, and $\Delta>0$, we see that (Eq. 27) can admit smooth solutions if $\mathcal{E}_\star^2<0$ (i.e., $M^2<\cos^2\theta$ and $\mathcal{E}_0^2>0$), and cusped solutions if $\mathcal{E}_\star^2>0$ (i.e., $M^2>\cos^2\theta$ and $\mathcal{E}_0<0$). In the latter case the denominator has a zero and produces an infinite wall in the Sagdeev potential. The corresponding solution is not analytic and has a cusp at the peak amplitude, but its profile differs from that of Porkolab and Goldman [14], and Shapiro [17]. It may also be of interest to note that these solutions can be written in closed but implicit forms. For the smooth solution, one can write

$$\mathcal{E}^2=\frac{\mathcal{E}_0^2\operatorname{csch}^2\psi}{\coth^2\psi-\mathcal{E}_0^2/\mathcal{E}_\star^2}, \tag{28}$$

where

$$\psi=\sqrt{\frac{-\mathcal{E}_0^2}{\mathcal{E}_\star^2}}\left(\arctan\sqrt{\frac{\mathcal{E}^2-\mathcal{E}_\star^2}{\mathcal{E}_0^2-\mathcal{E}^2}}-\frac{\pi}{2}\right)-\tilde{\eta},$$

which we note contains $\mathcal{E}^2$. For the cusped solution one can write

$$\mathcal{E}^2=-\frac{\mathcal{E}_0^2\operatorname{sech}^2\psi}{\tanh^2\psi-\mathcal{E}_0^2/\mathcal{E}_\star^2}, \tag{29}$$

where

$$\psi=\sqrt{\frac{-\mathcal{E}_0^2}{\mathcal{E}_\star^2}}\arctan\sqrt{\frac{\mathcal{E}_\star^2-\mathcal{E}^2}{\mathcal{E}^2-\mathcal{E}_0^2}}-\tilde{\eta},$$

and care should be taken in assembling the piecewise solutions. The transcendental solutions (28) and (29) may be verified by direct substitution.

V. DISCUSSIONS

In this paper we have extended the earlier investigations on Langmuir and upper-hybrid wave modulation to hf electron wave propagation at arbitrary angles to the external magnetic field. The generalized equations describing the evolution of the hf wave envelope and the lf electrostatic background density modulation are obtained. It is shown that the wave modulation is strongly propagation-angle dependent. For quasistationary propagation, new regimes of localized smooth and cusped solutions for the modulated wave envelope are found. The corresponding solitary waves are of small but finite amplitude and can propagate at sub and near-sonic speeds. Our results can be useful for more precise identification of the nonlinear waves and the corresponding density modulations in the data from the ionospheric and magnetospheric observations.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Project No. 19975043) and the Sonderforschungsbereich 191 Niedertemperaturplasmen (Germany).

[1] V.E. Zakharov, Zh. Éksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].

[2] K. Nishikawa, H. Hojo, and K. Mima, Phys. Rev. Lett. **33**, 148 (1974).

[3] V.I. Karpman, Phys. Scr. **11**, 263 (1975).

[4] H. Schamel, M.Y. Yu, and P.K. Shukla, Phys. Fluids **20**, 1286 (1977).

[5] B. Buti and M.Y. Yu, J. Plasma Phys. **26**, 309 (1981).

[6] N.N. Rao and R.K. Varma, J. Plasma Phys. **27**, 95 (1982).

[7] P. M. Bellan and K. Stasiewicz, Phys. Rev. Lett. **80**, 3523 (1998).

[8] R. Cesario, R. Bartiromo, A. Cardinali, F. Paoletti, V. Pericoli-Ridolfini, and R. Schubert, Nucl. Fusion **32**, 2127 (1992).

[9] G.V. Khazanov, T.E. Moore, E.N. Krivorutsky, J.L. Horwitz, and M.W. Liemohn, Geophys. Res. Lett. **23**, 797 (1996).

[10] H.L. Pecseli, K. Iranpour, O. Holter, B. Lybekk, J. Holtet, J. Trulsen, A. Eriksson, and B. Holback, J. Geophys. Res. **101**, 5299 (1996).

[11] K. Stasiewicz, B. Holback, V. Krasnoselskikh, M. Boehm, R. Boström, and P.M. Kintner, J. Geophys. Res. **101**, 21 515 (1996).

[12] J. Bonnell, P. Kintner, J.-E. Wahlund, and J.A. Holtet, J. Geophys. Res. **102**, 17 233 (1997)

[13] A.N. Kaufman and L. Stenflo, Phys. Scr. **11**, 169 (1975).

[14] M. Porkolab and M.V. Goldman, Phys. Fluids **19**, 872 (1976).

[15] M.Y. Yu and P.K. Shukla, J. Plasma Phys. **19**, 889 (1977).

[16] K.B. Dysthe, E. Mjølhus, H.L. Pécseli, and L. Stenflo, J. Plasma Phys. **20**, 1087 (1978).

[17] V.D. Shapiro, Phys. Rev. Lett. **81**, 3415 (1998).

[18] F. F. Chen, *Introduction to Plasma Physics* (Plenum, New York, 1974), Chap. 8.